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1. ABSTRACT OF THE ORIGINAL PROPOSAL

In this project the principal investigator and his co-workers wish to exploit their expertise in the theoretical analysis, particularly regarding superconvergence, and practical application of finite element methods to produce and analyse new finite element schemes for treating difficult linear and nonlinear problems of solid mechanics and fracture mechanics. The difficulties arise from the presence of effects such as material nonlinearities, nonhomogeneity, time dependence and stationary and moving cracks. In particular they wish to apply new retrieval techniques of finite elements to define new adaptive procedures for producing numerical solutions with enhanced properties for some problems of linear elasticity and linear elastic fracture, elasto-plasticity and nonlinear fracture, viscoelasticity and viscoelastic fracture. Contexts in which these new techniques will be applied include the deformation and fracture of timbers, and elasto-plastic deformation and fracture of metals with work hardening and the viscoelastic deformation and fracture of various polymers. Attention will also be given to some composite structures in these contexts. Whenever possible the numerical results will be compared with experimental results.

2. SUMMARY OF RESULTS ACHIEVED

(i) **Gradient recovery** techniques for finite element approximations to problems of linear elasticity and linear viscoelasticity were defined and **superconvergent** error estimates were derived. These results demonstrate that recovery techniques should routinely be used for finite element approximation of problems of this type, because they give increased accuracy and convergence rates at little extra cost, (Refs 2-6).

(ii) The **gradient recovery** techniques have proved effective for calculating J-integrals in linear elastic fracture.

(iii) Numerical algorithms for the deformation of quasistatic linear viscoelasticity were defined and applied to problems involving specific polymeric materials. Results showing the range of applicability of the **mathematical model** of viscoelasticity on which the algorithms are based were derived (Ref. 7). Theoretical error estimates for the numerical solution of the viscoelastic problems were derived; the principal investigator believes this to be the first time that this has been achieved for viscoelastic problems, (Ref 7).

(iv) Crack initiation and crack motion for a Mode I linear viscoelastic fracture problem were modelled using the numerical algorithms of (i) with a Barenblatt failure zone, (Refs 8, 9, 13).

(v) A variational inequality formulation in strain space, and its finite element discretisation, were defined for an **elastoplastic problem with hardening**. The algorithm was tested on a plate problem and the numerical results compare well with those of experiment, (Ref 11).

(vi) A finite volume method was proposed for modelling viscoelastic liquids under slow flow conditions. The numerical results compare well with those of experiment for the difficult case of Weissenberg number, (Ref 2).

All the above work has produced new numerical algorithms and codes which are available for use in modelling problems of elasticity, elastoplasticity and viscoelasticity. Confidence in the results has been achieved through the error analysis and through comparison with experiment.

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September 1989 - November 1992

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3. REVIEW OF WORK UNDERTAKEN

3.1 Linear Elasticity

The first task of this project was to extend the **superconvergence** results for the recovered gradients of piecewise linear finite element approximations to the solutions of Poisson problems, originally given in Ref 1, to the context of linear elasticity and linear elastic fracture

This task was achieved in a number of steps. Recovery techniques were first developed for **piecewise quadratic** finite element approximations to Poisson problems in **two dimensions**, and superconvergence results were established; see Refs 3 and 4. All the recovery techniques as above were then extended to the **three-dimensional** context using tetrahedral meshes

The next step involved the transfer of the finite element gradient recovery machinery as above to **linear elasticity** and again for superconvergent error estimates to be proved, see Ref 2. Of particular note is the fact that all these error estimates hold over subdomains of the problems and are therefore useful for problems having solutions of low regularity. In particular the recovery techniques can be used for problems of **linear elastic fracture**, where approximations to J-integrals have to be calculated; the stress intensity factor is used as the criterion for the onset of fracture in this context and this can be calculated via the path independent J-integral. The paths of the J-integrals can be taken in subdomains remote from the crack tips, where the recovered gradients are superconvergent and increased rates of convergence are achieved. Simultaneously, in order to improve global accuracy and convergence rates, specific activities such as mesh refinement may be undertaken local to the crack tips.

A criticism of superconvergent gradient recovery has always been that the proofs demand very regular meshes. The results obtained in the work of this project mean that fine regular meshes can now be concentrated over subdomains where major changes in the gradients of the problem solutions are taking place (e.g. stress concentrations) with superconvergence being obtained in these subdomains. Irregular meshes can be used elsewhere. Simultaneously work on less regular meshes was undertaken separately, and some superconvergent error estimates on quasiuniform mesh partitions of simplex elements were derived, see Ref 6

3.2 Time Dependent Problems, Viscoelasticity

Considerable effort was also put into the numerical solution of time-dependent problems. Linear and nonlinear **parabolic** problems were first treated, and for these **superconvergent** results for the recovered space gradients of discrete time/piecewise linear Galerkin in space approximations were proved, see Ref 5. This work extended the recovery concepts as above to the space/time context.

Problems of **viscoelasticity** were then treated and discrete time/Galerkin in space numerical schemes were proposed and implemented for the quasistatic context. Complete theoretical error analyses were derived for these methods, see Ref 7, this being the first time, to the authors' knowledge, that this had been done for problems of viscoelasticity. The estimates followed closely those for parabolic problems as above, and the parabolic superconvergence results were extended to the quasistatic viscoelastic case.

A feature of the work of Ref 7 is the application of the numerical algorithm to a specific problem involving the material nylon 66 "Maranyl", (registered trade mark) type A101, using experimental data, and comparing numerical with experimental results.. The numerical results demonstrate the

range of loadings and times over which the linear viscoelastic model is adequate for the material. Correspondingly they are valuable in that they indicate where a nonlinear model of the material behaviour is required.

For cases where the mathematical model is adequate, the algorithms of Ref 7 have been demonstrated to be effective numerical tools for quasistatic linear viscoelasticity.

3.3 Viscoelastic Fracture

The algorithms of Ref 7 were applied to the case of a crack in a viscoelastic material, and the modelling of crack initiation and crack motion for Mode I linear viscoelastic fracture was achieved. The model uses a Barenblatt type failure zone and a **crack opening displacement** (COD) criterion for crack initiation, see Refs 8, 9 , 10.

3.4 Elastoplasticity

Another area which was identified for analysis in the original proposal was that of elastoplastic deformation. For flow theory of plasticity a **variational inequality formulation in strain space** was derived and a finite element solution and algorithm using this for an elastoplastic problem with hardening was achieved. This work is described in Ref 11, where an example of a plate containing a hole in elastoplastic deformation was treated. The expansion of the plastic zone with increase of load is predicted, and the results agree well with experiments.

The algorithm led to a numerical tool for modelling this type of plasticity, which is currently being extended to nonlinear fracture.

3.5 Viscoelastic Liquids

The viscoelastic analysis of Section 3.2 was for solid materials. In nonisothermal problems the liquid and solid phase states, near melt temperature, exist together and interact on each other. It was therefore decided to undertake some numerical modelling of **viscoelastic liquids** in very slow viscoelastic flow conditions. A finite volume method in this context was proposed and was applied to a problem involving expansion, Ref 12. This produced very good numerical results, which again agree very well with those of experiment for quite high Weissenberg numbers. This work is very important in the context of polymeric extrusion processes and relates closely to the solid viscoelastic contexts described above.

4. OUTCOMES

All the above work constitutes steady advance in the treatment of problems of solid mechanics involving linear elastic, viscoelastic and elastoplastic deformation and also in the context of viscoelastic liquid flow. The proposer believes that the results obtained have been significant and that taken together they produce a very successful outcome for this project. As a result of this work the mechanisms of superconvergent recovery are now available for many problems of solid mechanics, the algorithm for linear viscoelastic fracture is available for the study of crack propagation in viscoelastic materials and the variational inequality formulation in strain space of elasto-plasticity has allowed a new algorithm for this type of deformation to be considered. All this is fully documented in the papers produced, Refs 2-13.

This work is currently being extended to allow **irregular mesh partitions** to be used for the recovery and for the recovered gradients to be exploited in adaptive schemes in order to obtain more accurate

numerical solutions and *a posteriori* error estimates. In the viscoelastic case the work of Reference 7 indicates the limitations of the linear viscoelastic model so that the possibilities of producing viscoelastic methods involving **nonlinear** constitutive relations are now being studied vigorously. It is expected that this will lead to fruitful research and methods which are applicable to a large range of polymeric materials. The work on crack propagation in viscoelastic materials also has important potential applications in the polymeric context, particularly for structural design.

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